Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 2
Due Monday January 28

4. Let G be a bipartite graph with bipartition X, Y. Hall's condition is $|T| \leq |N(T)|$ for all $T \subseteq X$. Prove that when $|X| = |Y|$ this is a necessary condition for G to have a perfect matching.

5. Let G be a bipartite graph with bipartition X, Y having $|X| = |Y|$. Prove that the sufficiency of Hall's condition for a perfect matching implies the sufficiency of the marriage condition. That is prove that the statement: (If G does not have a perfect matching then there is a $T \subseteq X$ such that $|T| > |N(T)|$) implies the statement (If G does not have a perfect matching then G has a vertex cover C with $|C| < n$).

6. Use induction to prove that the Fibonacci numbers satisfy $\sum_{i=0}^{n-1} F_{2i+1} = F_{2n}$.

7. Prove that the Arithmetic-Geometric mean inequality implies the Geometric-Harmonic mean inequality. That is, for positive numbers $y_1, y_2, \ldots, y_m$ the inequality
$\frac{y_1+y_2+\cdots+y_n}{n} \geq (y_1 y_2 \cdots y_n)^{1/n}$ implies $(y_1 y_2 \cdots y_n)^{1/n} \geq \frac{n}{\frac{1}{y_1}+\frac{1}{y_2}+\cdots+\frac{1}{y_n}}$.
Hint - think about reciprocals.

8. Derive a formula for $\sum_{i=1}^{n} i^3$ as follows. Note that $n^4 = \sum_{i=1}^{n} (i^4 - (i-1)^4)$ and then expand the $(i-1)^4$ term. The expression being summed now has terms involving i, i^2, i^3. Use known formulas for $\sum_{i=1}^{n} i^2$, $\sum_{i=1}^{n} i$, $\sum_{i=1}^{n} 1$ to get a formula for $\sum_{i=1}^{n} i^3$.