

Math 163 Introductory Seminar - Lehigh University - Spring 2008 - Assignment 4  
Due Monday February 11

11. We can consider perfect matchings in graphs in general, not just bipartite graphs. For this we use Tutte's condition:  $odd(G-S) \leq |S|$  for all  $S \subseteq V$ . Do not worry what  $odd(G-S)$  means or what the condition is about for now. It is known that Tutte's condition is necessary and sufficient for a graph  $G$  to have a perfect matching.

(a) State a condition equivalent to: Tutte's condition is a necessary condition for  $G$  to have a perfect matching.

(b) State the contrapositive of the statement in (a).

(c) State a condition equivalent to: Tutte's condition is a sufficient condition for  $G$  to have a perfect matching.

(d) State the contrapositive of the statement in (a).

For (a) and (c) one of your statements should be of the form 'If  $G$  has a perfect matching then ...' and the other should be of the form 'If ... then  $G$  has a perfect matching.' The '...' should be statements using terms like 'for some  $S \subseteq V$ ', 'for all  $S \subseteq V$ ',  $|S|$ ,  $odd(G-S)$ ,  $\leq$  and  $>$ . For (b) and (d) you should give 'if ... then' statements using terms like those in the previous sentence (and you should not use the word 'not') and ' $G$  does not have a perfect matching'.